In-class exercise

Instructions

- **Don't look at the solution yet!** This is for your benefit.
- This exercise must be submitted within 48 hours of the lecture in which it was given.
- As long as you do the exercise on time, you get full credit—your performance does not matter.
- Without looking at the solution, take 5 minutes to try to solve the exercise.
- Pre-assessment: Write down how correct you think your answer is, from 0 to 100%.
- Post-assessment: Now, study the solution and give yourself a "grade" from 0 to 100%.
- Submit your work on the course website, including the pre- and post- assessments.

Exercise

Consider the following Exponential model for an observation x:

$$p(x|a,b) = ab\exp(-abx)\mathbb{1}(x > 0)$$

and suppose the prior is

$$p(a,b) = \exp(-a-b)\mathbb{1}(a,b > 0).$$

You want to sample from the posterior $p(a,b|x)$. Find the conditional distributions needed for implementing a Gibbs sampler.

Solution

The Gibbs sampler consists of alternately sampling from $a|b, x$ and $b|a, x$. First note that the joint p.d.f. is

$$p(x, a, b) = ab \exp(-abx - a - b)\mathbb{1}(a, b, x > 0).$$

Thus,

$$p(a|b, x) \underset{a}{\propto} p(x, a, b) \underset{a}{\propto} a \exp(-abx - a)\mathbb{1}(a > 0) = a \exp(-(bx + 1)a)\mathbb{1}(a > 0) \underset{a}{\propto} \text{Gamma}(a \mid 2,\, bx + 1).$$

Therefore, $p(a|b, x) = \text{Gamma}(a \mid 2,\, bx + 1)$ and by symmetry, $p(b|a, x) = \text{Gamma}(b \mid 2,\, ax + 1)$.