In-class exercise

Instructions

- **Don't look at the solution yet!** This is for your benefit.
- This exercise must be submitted within 48 hours of the lecture in which it was given.
- As long as you do the exercise on time, you get full credit—your performance does not matter.
- Without looking at the solution, take 5 minutes to try to solve the exercise.
- Pre-assessment: Write down how correct you think your answer is, from 0 to 100%.
- Post-assessment: Now, study the solution and give yourself a "grade" from 0 to 100%.
- Submit your work on the course website, including the pre- and post- assessments.

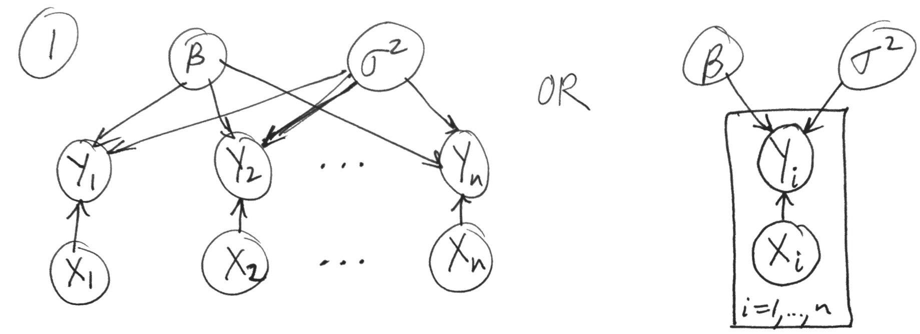
Exercise

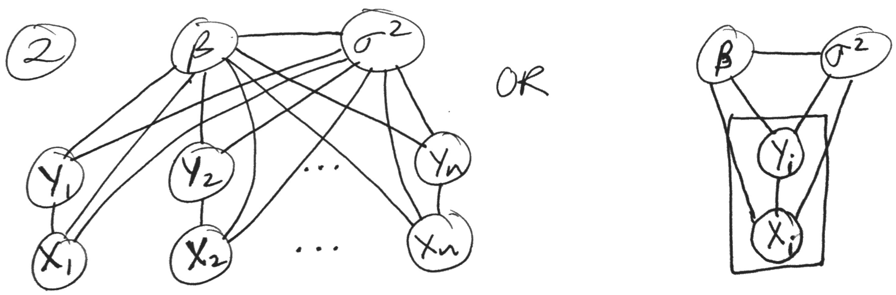
Consider the following model:

$$
\begin{aligned}
&\beta \sim \mathcal{N}(\mu_0, \sigma_0^2) \\
&\sigma^2 \sim \text{InvGamma}(a, b) \\
&X_1, \ldots, X_n \overset{\text{iid}}{\sim} \mathcal{N}(0, 1) \\
&Y_i | \beta, \sigma^2, x_{1:n} \sim \mathcal{N}(\beta x_i, \sigma^2) \text{ independently for } i = 1, \ldots, n.
\end{aligned}
$$

1. Draw the (natural) directed graphical model.
2. Draw the corresponding moral graph.
3. (a) Are β and σ^2 independent?
   (b) Are β and σ^2 independent given $X_{1:n}$ and $Y_{1:n}$?
   (c) Are Y_1 and Y_2 independent?
   (d) Are Y_1 and Y_2 independent given β and σ^2?

Solution





3. It is important to note that the graphical model can tell you that two variables are conditionally independent, but cannot tell you that they are not conditional independent (i.e., it cannot tell you that they are conditionally dependent).

   (a) Yes, by construction. (Write down the joint pdf to convince yourself.)

   (b) No, since, for example, $p(\beta|\sigma^2, x_{1:n}, y_{1:n})$ depends on σ^2. (Derive this conditional distribution to convince yourself.)

   (c) No, since, for example, the posterior predictive $p(y_2|y_1)$ depends on y_1. (We can tell that this will be the case since knowledge of y_1 gives us information about σ^2 — e.g., a large value of y_1 indicates a large σ — which, in turn, gives us information about y_2.)

   (d) Yes. In the moral graph, every path from Y_1 to Y_2 passes through β or σ^2.