In-class exercise

Instructions

- **Don't look at the solution yet!** This is for your benefit.
- This exercise must be submitted within 48 hours of the lecture in which it was given.
- As long as you do the exercise on time, you get full credit—your performance does not matter.
- Without looking at the solution, take 5 minutes to try to solve the exercise.
- Pre-assessment: Write down how correct you think your answer is, from 0 to 100%.
- Post-assessment: Now, study the solution and give yourself a "grade" from 0 to 100%.
- Submit your work on the course website, including the pre- and post- assessments.

Exercise

1. What is the multivariate normal density $\mathcal{N}(x|\mu, C)$?
2. Suppose X, Y are two real-valued random variables.
   (a) What is the formula for the covariance $\mathrm{Cov}(X, Y)$?
   (b) What is the formula for the correlation $\rho(X, Y)$?
3. Suppose you can generate $\mathcal{N}(0, 1)$ random variables. Give an explicit formula for generating a sample from $\mathcal{N}(\mu, C)$ where $C = AA^{\intercal}$.

Solution

1.

$$\frac{1}{(2\pi)^{d/2}|C|^{1/2}} \exp\big(-\tfrac{1}{2}(x-\mu)^{\mathsf{T}} C^{-1}(x-\mu)\big)$$

2. (a) $\mathrm{Cov}(X,Y) = \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y) = \mathbb{E}\big((X-\mathbb{E}X)(Y-\mathbb{E}Y)\big)$

   (b) $\rho(X,Y) = \mathrm{Cov}(X,Y)/(\sigma(X)\sigma(Y))$

3. If $Z_1, \ldots, Z_d \overset{\text{iid}}{\sim} \mathcal{N}(0,1)$ and $Z = (Z_1, \ldots, Z_d)^{\mathsf{T}}$, then $AZ + \mu \sim \mathcal{N}(\mu, AA^{\mathsf{T}})$.