In-class exercise

Instructions

- **Don't look at the solution yet!** This is for your benefit.
- This exercise must be submitted within 48 hours of the lecture in which it was given.
- As long as you do the exercise on time, you get full credit—your performance does not matter.
- Without looking at the solution, take 5 minutes to try to solve the exercise.
- Pre-assessment: Write down how correct you think your answer is, from 0 to 100%.
- Post-assessment: Now, study the solution and give yourself a "grade" from 0 to 100%.
- Submit your work on the course website, including the pre- and post- assessments.

Exercise

Show that if $p(\theta)$ and $q(\theta)$ are both p.d.f.s. and

$$p(\theta) \propto q(\theta)$$

then

$$p(\theta) = q(\theta)$$

for all θ.

Solution

The general definition of proportionality is that two functions $f(x)$ and $g(x)$ are proportional if there exist constants a, b which are not both equal to 0 (i.e., either a is nonzero, or b is nonzero, or both of them are nonzero) such that

$$af(x) = bg(x)$$

for all x. Applying this to the case of p and q above, there are some a, b, not both zero, such that

$$ap(\theta) = bq(\theta) \tag{0.1}$$

for all θ, thus

$$a = a\int p(\theta)d\theta = \int ap(\theta)d\theta = \int bq(\theta)d\theta = b\int q(\theta)d\theta = b.$$

Therefore, $a = b$, so both a and b are nonzero, and cancelling a with b in Equation 0.1 shows that $p(\theta) = q(\theta)$ for all θ.

Note: When we are dealing with p.d.f.s or p.m.f.s, both a and b must be nonzero, so the general definition is equivalent to: $f(x)$ and $g(x)$ are proportional if there exists c such that

$$f(x) = cg(x)$$

for all x.