Homework 9

The full chapter 8 from Bishop's book has now been posted under Lessons. Note: DGM means "directed graphical model". A directed graphical model must always use a directed *acyclic* graph.

1. Bishop exercise 8.1. Note: By "correctly normalized", he means that it sums (or integrates) to 1.

2. Bishop exercise 8.8.

3. Give an example of a DGM on four variables W, X, Y, Z that is respected by any distribution on W, X, Y, Z. In other words, no matter what the joint distribution of W, X, Y, Z is, it will factor in accordance with your graph. Justify your answer.

4. Give an example of a DGM on four variables W, X, Y, Z such that $X \perp Y \mid W$, and this property can be determined from the DGM, but cannot be determined from the moral graph. Justify your answer.

5. Consider the model from Lab 7.

   (a) Draw the natural DGM for this model. (Here, by natural, I mean the one resulting from the factorization corresponding to the way the model is specified.) Do not include the ϵ's in your DGM. Represent the Y's as normal given the β's and τ (and the x's).

   (b) Draw the resulting moral graph.

   (c) Using the moral graph, show that (μ_0, τ_0) is conditionally independent of everything else given $\beta_{01}, \ldots, \beta_{0n}$.

   (d) Using the moral graph, show that the observations Y for patient 1 are conditionally independent of those for patient 2 given $\mu_0, \tau_0, \mu_1, \tau_1$, and τ.