Lab 1 Solutions

(Statistical Learning, BST 263)

Practice problems (Individual activity – 10 minutes)

(Problems 1 and 2 are to be handed in on paper during class.)

1. From the hint,
$$f(x) = \sum_{i=1}^{m} \Big(\sum_{j=1}^{n} A_{ij} x_j \Big)^2.$$
Note that
$$\frac{\partial}{\partial x_k} \sum_{j=1}^{n} A_{ij} x_j = A_{ik}$$
since $\frac{\partial}{\partial x_k} x_j = \mathrm{I}(j = k)$. Thus, by the chain rule,
$$\big(\nabla f(x)\big)_k = \frac{\partial}{\partial x_k} f(x) = \sum_{i=1}^{m} \frac{\partial}{\partial x_k} \Big(\sum_{j=1}^{n} A_{ij} x_j \Big)^2 = \sum_{i=1}^{m} 2 A_{ik} \Big(\sum_{j=1}^{n} A_{ij} x_j \Big).$$
This can be succinctly written in linear algebra notation as
$$\nabla f(x) = 2 A^{\mathsf{T}} A x$$
since $A_{ik} = (A^{\mathsf{T}})_{ki}$.

2. Taking the partial derivative of $\frac{\partial}{\partial x_k} f(x)$ with respect to x_ℓ gives us
$$\begin{aligned} \big(\nabla^2 f(x)\big)_{k\ell} &= \frac{\partial}{\partial x_\ell} \frac{\partial}{\partial x_k} f(x) = \frac{\partial}{\partial x_\ell} \sum_{i=1}^{m} 2 A_{ik} \Big(\sum_{j=1}^{n} A_{ij} x_j \Big) \\ &= \sum_{i=1}^{m} 2 A_{ik} \frac{\partial}{\partial x_\ell} \Big(\sum_{j=1}^{n} A_{ij} x_j \Big) = \sum_{i=1}^{m} 2 A_{ik} A_{i\ell}. \end{aligned}$$
This can also be succinctly written in linear algebra notation:
$$\nabla^2 f(x) = 2 A^{\mathsf{T}} A$$
since $A_{ik} = (A^{\mathsf{T}})_{ki}$.

KNN and cancer classification (Team activity)

See `lab-1-solutions.r` under Files/Labs on Canvas.